

NAG Toolbox for MATLAB

f07fj

1 Purpose

f07fj computes the inverse of a real symmetric positive-definite matrix A , where A has been factorized by f07fd.

2 Syntax

```
[a, info] = f07fj(uplo, a, 'n', n)
```

3 Description

f07fj is used to compute the inverse of a real symmetric positive-definite matrix A , the function must be preceded by a call to f07fd, which computes the Cholesky factorization of A .

If **uplo** = 'U', $A = U^T U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-T}$.

If **uplo** = 'L', $A = LL^T$ and A^{-1} is computed by first inverting L and then forming $L^{-T}(L^{-1})$.

4 References

Du Croz J J and Higham N J 1992 Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = LL^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper triangular matrix U if **uplo** = 'U' or the lower triangular matrix L if **uplo** = 'L', as returned by f07fd.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

U contains the upper triangle of A^{-1} if **uplo** = 'U'; L contains the lower triangle of A^{-1} if **uplo** = 'L'.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the i th diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n , ϵ is the *machine precision* and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this function is f07fw.

9 Example

```
uplo = 'L';
a = [2.039607805437114, 0, 0, 0;
     -1.529705854077835, 1.640121946685673, 0, 0;
     0.2745625891934577, -0.2499814119483738, 0.7887488055748053, 0;
     -0.04902903378454601, 0.6737303907389101, 0.6616575633742563,
```

```
0.5346894269298688];  
[aOut, info] = f07fj(uplo, a)
```

```
aOut =  
    0.6995         0         0         0  
    0.7769    1.4239         0         0  
    0.7508    1.8255    4.0688         0  
   -0.9340   -1.8841   -2.9342    3.4978  
info =  
      0
```
